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University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

FINAL EXAMINATION
MATA32 - Calculus for Management I

Examiners: R. Grinnell

Date: April 27, 2011
Time: 2:00 pm
Duration: 3 hours

Provide the following information:

Lastname (PRINT): ______________________________

Given Name(s) (PRINT): ______________________________

Student Number : ______________________________

Signature: ______________________________

Read these instructions:

1. This examination has 13 numbered pages. It is your responsibility to ensure that at the beginning of the exam, all of these pages are included.
2. If you need extra answer space, use the back of a page or page 13. Clearly indicate the location of your continuing work. You may write in pencil, pen, or other ink.
3. You may use one standard hand-held calculator (graphing capability is permitted). All other electronic devices (e.g. cell phone, smart phone, i-pod), extra paper, notes, textbooks, and backpacks are forbidden at your workspace.
4. If you have brought a cell/smart phone into the GYM, it must be turned off and left at the front of the GYM.

Print letters for the Multiple Choice Questions in these boxes:

1	2	3	4	5	6	7	8	9	10

Do not write anything in the boxes below.

A	1	2	3	4	5	6	7	8	**TOTAL**
40	16	16	13	12	15	12	18	8	150

The following may be helpful:

$$S = P(1+r)^n \qquad S = Pe^{rt} \qquad S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$S = R\left[\frac{(1+r)^{n+1} - 1}{r}\right] - R \qquad A = R + R\left[\frac{1-(1+r)^{-n+1}}{r}\right]$$

Profit = Revenue - Cost $\qquad NPV = (\sum PV) - \text{Initial} \qquad \eta = \frac{p/q}{dp/dq}$

Part A - Multiple Choice Questions For each of the following, **clearly print the letter of the answer you think is most correct in the boxes on page 1**. Each right answer earns 4 points and no answer/wrong answers earn 0 points. No justification is required.

1. The function $g(x) = -2x^3 - x^2$ has

   (A) a relative maximum at 0 (B) a relative minimum at 0
   (C) an absolute maximum at 0 (D) an absolute minimum at 0
   (E) both A and C (F) both (B) and D (G) no extrema at 0

2. The area of the region lying between the curve $y = x^2 + 4x$ and the lines $x = 0$, $y = -1$, and $x = 2$ equals

   (A) 6 (B) 26/3 (C) 32/3 (D) 38/3 (E) 14 (F) a number not in (A) - (E)

3. Assume y is defined implicitly as a function of x by the equation $xy^2 + 4x^2y = 4x + 8$. Then y' evaluated at $x = 1$ and $y = 2$ is

(A) -2 (B) 2 (C) 0 (D) $-8/5$ (E) a number not in (A) - (D)

4. Assume the average revenue obtained by selling x units is $a(x) = \dfrac{x^2 + 5x + 2}{x + 3}$. Then the marginal revenue from selling 7 units is

(A) 1.04 (B) 2.19 (C) 35.48 (D) 15.88 (E) a number not in (A) - (D)

5. If $y' = 3\sqrt{x}$ and $y(1) = 5$, then $y(4)$ equals

(A) 13 (B) 16 (C) 18 (D) 19 (E) 20 (F) a number not in (A) - (E)

6. The maximum amount of compound interest (expressed as a percentage to one decimal) that can be earned on a deposit of $\$D$ over 25 years at 3% APR is

(A) 104.3 (B) 108.6 (C) 109 (D) 111.7 (E) a number not in (A) - (D)
(F) uncertain because we are not given enough information.

7. If $p = -4q + 96$ is a demand function (p is price, q is quantity) where $0 < q < 24$ then we have unit elasticity at

(A) $q = 6$ (B) $q = 8$ (C) $q = 12$ (D) $q = 16$ (E) a value of q not in (A) - (D).

8. The future value of \$812.32 in 57 months at 4.4% APR compounding quarterly is (to the nearest dollar rounded up)

(A) \$1,000 (B) \$1,072 (C) \$1,100 (D) \$1,153 (E) a number not in (A) - (D).

9. If $y = (2x)^x$ then $y'(1)$ equals

(A) 1 (B) $ln(4)$ (C) $1 + ln(4)$ (D) $2 + ln(4)$ (E) a number not in (A) - (D).

10. Exactly how many of the following statements are always true?

- If $G_1(x)$ and $G_2(x)$ are antiderivatives of a function $g(x)$, then $G_1(x) = G_2(x)$ for all x in the domain of g.
- The definite integral of a function f is a function F such that $\int f(x)\,dx = F(x) + C$ where C is an arbitray constant.
- For a continuous function h, the Fundamental Theorem of Calculus states that $\int_a^b h(x)\,dx$ is defined to be $H(b) - H(a)$ where H is any antiderivative of h.
- If f is continuous for all real x, then $\frac{d}{dx}\left(\int f(x)\,dx\right) = f(x)$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

(Be sure you have printed the letters for your answers in the boxes on page 1)

Part B - Full Solution Questions Write clear, full solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32.

1. Find the following integrals. Express your answers in terms of mathematical constants, not decimals provided by a calculator.

   (a) $\int 3x^5 (x^3 + 8)^{1/3} \, dx$ [8 points]

   (b) $\int_1^e x^4 ln(x) \, dx$ [8 points]

2. (a) Find the exact value of $u'(1)$ where $u(x) = \dfrac{2^{(x^3+x)}\, x^5}{\sqrt{x^2+4}}$

("Exact value" means an answer in terms of mathematical constants, not decimals provided by a calculator)

[8 points]

(b) Find the absolute extrema (and where they occur) of the function $f(x) = 2x^3 + x^2 - 4x$ where $x \in [0, 2]$.

[8 points]

3. A rectangular plot of land has area $A > 0$ square metres and a special fence is required to completely enclose it. The length (i.e. horizontal) dimension of the fence is x metres and the width (i.e. vertical) dimension is y metres. The length fence material costs $\$k$ per metre and the width fence material costs $\$m$ per metre. Find the dimensions x and y of the plot of land that results in a fence of absolute minimum cost and state this minimum cost. Your answers should be in terms of A, k, and m. Be sure to verify that your answers actually do give the least fence cost.

[13 points]

4. In the space below, make axes that extend to ± 5 along the x-axis and from -2 to 5 along the y-axis. Then sketch the graph of a function $y = f(x)$ having **all** of the following properties in (i) - (v) given below. Points are awarded for accuracy and neatness. Note that there are many quite-different looking graphs for functions that satisfy all of the properties below.

[12 points]

(i) f is continuous for $x \in \mathbf{R},\ x \neq 0$, and $f(0)$ is undefined.

(ii) $f(\pm 1) = 1$ and $(2, 2)$ is a point of inflection.

(iii) $f''(x) > 0$ for all $x \in (-\infty, -1) \cup (0, 2)$.

(iv) $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 3$.

(v) the sign of $f'(x)$ is the same as the sign of $g(x) = (x - 1)(x^2 - 1)^2$ for $x \in \mathbf{R},\ x \neq 0$. (The phrase "the sign of" refers to positiveness, negativeness, and zero.)

5. (a) In this question all money is in units of $\$1,000$. An initial investment of 110 guarantees the following four cash flows from a business venture for the next six years:

Year $\rightarrow$	2	4	5	6
Cash Flow $\rightarrow$	F	32	38	40

Interest is 3.8% APR compounding semi-annually and all amounts in the cash flow schedule are payed at the end of the indicated year. Find the least value of F above (rounded up to the nearest \$100) so that the business venture will be profitable.

[8 points]

(b) Assume Accounts 1 and 2 are empty to begin with. Suppose you deposit $\$P$ into an ordinary annuity in Account 1 at the end of every 3 months for the next 10 years (thus you make 40 deposits). Interest is 6% APR compounding quarterly. Immediately after the last payment, the accumulated amount in Account 1 is transferred into Account 2 where it is left for 30 years with interest at 4% compounding semi-annually.

Find the least value of P (rounded up to the nearest dollar) so that the final amount in Account 2 is 1/2 million dollars. Use your rounded value for P to find the percentage (rounded to one decimal) of this 1/2 million dollars that is generated by interest only. [7 points]

6. Find the equations of the tangent lines to the curve $y = x^2 - 2x - 1$ that pass through the point $(-3, 5)$. [12 points]

7. In all of this question $\mathcal{R}$ represents the enclosed region bounded between the curves $y^2 + x = 4$ and $x + y + 2 = 0$.

   (a) Give a labeled sketch that clearly shows $\mathcal{R}$ and the intercepts/intersection points of the boundary curves. [5 points]

   (b) Find the area of $\mathcal{R}$. [9 points]

Question 7 continued.

(c) Calculate the percentage of the area of $\mathcal{R}$ that lies in the first quadrant (i.e. $x, y \geq 0$). [4 points]

8. Find the cubic polynomial Q (i.e. degree is 3) such that:
$Q(1) = -4,\ Q^{(1)}(1) = -15,\ Q^{(2)}(1) = -20,$ and $Q^{(3)}(1) = -24.$ [8 points]

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