

University of Toronto Scarborough
Department of Computer & Mathematical Sciences

Midterm Test
MATA32H – Calculus for Management I

Examiner: E. Moore

Date: March 3, 2018

Start time: 9:00am

Duration: 110 minutes

1. **[5 points]** You have just won a lottery and you are given two payment options:
1. \$850,000 now
 - or 2. 10 annual payments of \$100,000, with the first payment now.
- If the effective interest rate is 4%, determine which is the better option.

2. **[13 points]**

- (a) Find the following limits, if they exist, You may use the symbols ∞ or $-\infty$ when needed. Always provide justification when appropriate. (The use of l'Hôpital's Rule will earn no credit.)

i. $\lim_{x \rightarrow 0} \frac{\sqrt[5]{1+3x} - 1}{x}$.

ii. $\lim_{s \rightarrow 2} \frac{s^2 + s - 6}{s^3 - 6s^2 + 8s}$.

(b) Let $f(x) = \begin{cases} \frac{1-x^3}{1-\sqrt{x}} & , \text{ for } x \neq 1 \\ 6 & , \text{ for } x = 1 \end{cases}$.

Determine if $f(x)$ is continuous at $x = 1$.

3. **[12 points]** Let $f(x) = \frac{8x}{1-3x}$.

- (a) Find the point(s) on the graph of $y = f(x)$ where the tangent line is parallel to the line $y = 2x - 7$.
- (b) Use the definition of derivative ("first principles") to find $f'(x)$.

4. **[10 points]**

- (a) Let $f(u) = u^3 - 3u^2 + 2u + 1$ where $u = u(x) = 2x^2 + e^{x-1}$. Use the Chain rule to find the value of $\frac{df}{dx}$ when $x = 1$.

- (b) Find $f'(x)$, in fully factored form, when $f(x) = (x^2 + 2x - 1)^3 (5x^3 + 1)^2$.

5. [10 points] Let $y = f(x)$ be defined implicitly by the expression

$$y^3 + xy^2 + xy + x^2 = 4.$$

- (a) Find the equation of the tangent line at $(1, 1)$.
- (b) Determine where the tangent line from part (a) crosses the x -axis or show that they do not meet.
6. [10 points] Let $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x$.
- (a) Find the critical points of $y = f(x)$.
(Hint: Is $x + 2$ a factor of $f'(x)$?)
- (b) Determine the intervals of increase and decrease and find relative extrema. (A sign chart is required.)
- (c) Find any absolute extrema that may exist.
7. [8 points] Suppose that a country's consumption function is given by

$$C = \frac{10\sqrt{I} + 0.7\sqrt{I^3} - 0.2I}{\sqrt{I}}$$

where C and I are expressed in billions of dollars.

- (a) Find the marginal propensity to save when income is \$25 billion.
- (b) Determine the relative rate of change of C with respect to I when income is \$25 billion.
8. [7 points] You are thinking about a trip after graduation so you just opened a new savings account and made a \$500 deposit. Over the next 5 months you add \$50 at the end of each month. Starting in month 6, you are able to add \$100 at the end of each month for the next 3 years. If interest is at a nominal rate of 3%, compounded monthly, how much will be in the account after the final deposit? Please round your answer to the nearest dollar.
9. [15 points] A total debt of \$7500 due in 2 years and \$2500 due in 6 years is to be repaid in the following way:
An initial payment now and 3 subsequent annual payments in 3 years, 4 years and 5 years, with each payment being \$500 more than the previous payment. Interest is at a nominal rate of 4.8% compounded monthly. Find the amount of each payment rounded up to the nearest dollar.
(A complete answer requires a money-time line and an equation of value.)
10. [15 points] Let $q > 0$ be the number of units and let $p = p(q) = \frac{450}{q+3}$ be the demand function.
- (a) Find the marginal revenue when $q = 9$.
- (b) Let $c = c(q)$ be the cost function. If the marginal cost of 9 units is 5 and the average cost of 9 units is 15, estimate the profit when 10 units are sold.
- (c) Determine those q ($q > 0$) for which the demand is elastic.