University of Toronto at Scarborough
Department of Computer & Mathematical Sciences

MATA33S **Assignment 8** **Winter 2018**

Work on this assignment in preparation for Quiz 5 which is in Week 11 (Friday March 23 - Thursday March 29). Also see the Note below.

Study: Sections 17.4 and 17.5 for this Assignment 8. Read ahead in Section 17.6 on Optimization of Functions of Several Variables.

Terminology and Concepts to Learn: (Section 17.4) higher-order partial derivatives, mixed partial derivatives, equality of mixed partial derivatives (Section 17.5) the chain rule for functions of several variables.

Problems:

1. Section 17.4, Pages 764 - 765 # 1, 2, 5, 9, 10, 16, 17, 19, 20, 22 - 24.
2. Section 17.5, Page 768 # 1 - 4, 9 - 13, 17, 20.
3. Page 795 # 18
4. For the following two functions find all point(s) (x, y) such that $f_x(x, y) = f_y(x, y) = 0$
   (a) $f(x, y) = x^2 + 2y^2 - x^2y$
   (b) $f(x, y) = xy + \dfrac{a^3}{x} + \dfrac{b^3}{y}$ where a and b are non-zero constants.
5. Verify that the function $z = xe^y + ye^x$ is a solution to the equation $z_{xxx} + z_{yyy} = xz_{xyy} + yz_{xxy}$
6. Throughout this problem consider n real variables $x_1, x_2, ..., x_n$ and let $u(x_1, x_2, ..., x_n) = \sum_{k=1}^{n} a_k x_k$ where a_k is a constant for each $k = 1, 2, ..., n$ and $\sum_{k=1}^{n} a_k^2 = 1$.
   Let $z = e^{u(x_1, x_2, ..., x_n)}$ and verify that $\sum_{k=1}^{n} \dfrac{\partial^2 z}{\partial x_k^2} = z$.
7. Let $z = x^2 + xy + y^2$, $x = s + t$, and $y = st$. Find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ two ways:
   (a) By first substituting x and y as functions of s and t and differentiating directly.
   (b) By the chain rule.
8. Repeat problem 7 for the functions $z = \dfrac{x}{y}$, $x = se^t$, and $y = 1 + se^{-t}$

Note: Quiz 5 (the last quiz) is in Week 5 (Friday March 23 - Thursday March 29). It will cover Section 17.4 from Assignment 7, all of this Assignment 8, and perhaps some aspects of Section 17.6 (which will be in Assignment 9). In addition to working on this Assignment 8, you should read ahead into Section 17.6. The MATA33S home page posting called "A33 Hessian Optimization" should also be studied.