A multinational company produces products x, y and z at plants p_1 and p_2 using raw materials a, b, c and d. The number of units of each raw material to produce 1 unit of each product is given by

	a	b	c	d
x	11	20	5	6
y	2	1	18	6
z	1	2	5	0

The cost per unit of raw materials delivered to each plant is given by

	p_1	p_2
a	3	1
b	2	2
c	1	2
d	1	1

.

Find the cost of producing each product at each plant.

Elementary operations

(1) interchange two equations

(2) multiply one equation by a nonzero constant

(3) add a constant multiple of the sides of one equation to the corresponding sides of another equation

Performing one of these elementary operations on a system of linear equations leaves the solution unchanged.

Given the system of linear equations

$$\begin{array}{ccccccc} A_{11}x_1 & +A_{12}x_2 & +\cdots & +A_{1n}x_n & = & B_1 \\ & & \vdots & & & \\ A_{m1}x_1 & +A_{m2}x_2 & +\cdots & +A_{mn}x_n & = & B_m \end{array}$$

the matrix $\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ & \vdots & & \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix}$ is called the **coefficient matrix** of the system and the matrix

$\left[\begin{array}{cccc|c} A_{11} & A_{12} & \cdots & A_{1n} & B_1 \\ & & \vdots & & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} & B_m \end{array}\right]$ is called the **augmented coefficient matrix** of the system.

Elementary row operations

1. interchanging two rows of a matrix

2. multiplying a row of a matrix by a nonzero number

3. adding a multiple of one row of a matrix to a different row of that matrix

Whenever one matrix can be obtained from another by elementary row operations we say the matrices are **equivalent**.

Note that the elementary row operations correspond to the elementary operations used on systems of linear equations.

Notation	Corresponding row operation
$R_i \longleftrightarrow R_j$	interchange rows R_i and R_j
$k\,R_i$	multiply row R_i by nonzero constant k
$k\,R_i + R_j$	add k times row R_i to row R_j (leaves row R_i unchanged)