**University of Toronto at Scarborough**
**Department of Computer and Mathematical Sciences, Mathematics**

MAT C34F 2019/2020

Problem Set #1

Due date: Thursday, September 12, 2019 at the beginning of class

(1) Prove that

$$f(z) = \sqrt{\mathrm{Re}(z)\mathrm{Im}(z)}$$

satisfies the Cauchy-Riemann equations at $z = 0$ but is not differentiable there.

(2) Prove that, for $z \in \mathbb{C}$,

$$|z| \leq |\mathrm{Re}(z)| + |\mathrm{Im}(z)| \leq \sqrt{2}|z|.$$

(3) Prove that for $z, w \in \mathbb{C}$,

$$\mathrm{Re}\left(\frac{w+z}{w-z}\right) = \frac{|w|^2 - |z|^2}{|w-z|^2}.$$

(4) Find the real and imaginary parts of the following functions as functions of $x$ and $y$:
  (a) $z^3$
  (b) $(z + z^{-1})$ $(z \neq 0)$
  (c) $\frac{1}{1-z}$ $(z \neq 1)$

(5) Show that $f(z) = \bar{z}$ and $g(z) = \mathrm{Im}(z)$ do not satisfy the Cauchy-Riemann equations.

(6) Which of the following is differentiable at $z = 0$? Give a proof or a counterexample.
  (a) $|z|^4$
  (b) $\mathrm{Re}(z) + \mathrm{Im}(z)$
  (c) $\mathrm{Re}(z)\mathrm{Im}(z)$