University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

MAT C34F 2013/14

Problem Set #3

Due date: Thurs Oct 10, 2013 at the beginning of class

1. Find the value of the integral of $g(z)$ around the circle $|z-i|=2$ in the positive sense when
$$g(z)=\frac{1}{z^2+4}.$$

2. Find an expansion $f(z)=\sum_{n=0}^{\infty} c_n(z-a)^n$ valid in the disk $D(a;r)$ when $f(z)=\sin^2 z$.

3. Is it possible to construct a holomorphic function on $D(0,1)$ such that $f(1/n)=z_n$ when $z_n=0$ for n even and $z_n=1/n$ when n is odd?

4. (a) Let f be holomorphic on $D(0;1)$.Prove that g defined by
$$g(z)=f(z)-f_2(z)$$
where $f_2(z)$ is the complex conjugate of $f(-\bar{z})$, is holomorphic in $D(0;1)$.

   (b) Now suppose f takes real values on the imaginary axis. Prove that for $x+iy\in D(0;1)$
$$u(x,y)=u(-x,y)$$
$$v(x,y)=-v(-x,y)$$
where u and v denote the real and imaginary parts of f.

5. Let G be the square region $\{z:|\mathrm{Re}(z)|<1,|\mathrm{Im}(z)|<1|\}$. Suppose f is continuous in $\bar{G}$, holomorphic in G, and such that $f(z)=0$ when $\mathrm{Re}(z)=1$. By considering g defined by $g(z)=f(z)f(iz)f(-z)f(-iz)$, prove that f is zero everywhere in $\bar{G}$.

6. Assume $|a|\neq 1$ and $|b|\neq 1$. Evaluate $\int_{\gamma(0;1)}\frac{1}{(z-a)(z-b)}$

   Evaluate for : Case 1 $|a|<1,|b|<1$,

   Case 2 $|a|<1,|b|>1$ or $|a|>1,|b|<1$

   Case 3 $|a|>1,|b|>1$