CSC B36 Additional Notes

common ways to write statements in English

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⋆ Introduction

English is a very rich language. Most statements can be expressed in a wide variety of ways, sometimes with subtle nuances in meaning. Mathematics and computer science are precise fields of study. Ideas must be expressed accurately. Statements must be made without ambiguity. Not surprisingly then, mathematicians and computer scientists have used many different ways to express their ideas in English. Here are some common ways for saying some common ideas.

◦ Implications

An implication is a statement of the form "if P then Q", where P and Q are propositions. A proposition is a statement that is true or false (but not both). Using mathematical notation, we would write something like "$P \rightarrow Q$". For our purposes here, we will use the following propositions.

P: Monty (a prize winning cucumber from a local garden) is boiled.
Q: Monty tastes yucky.

Here are some common ways to say $P \rightarrow Q$.

(a) *If Monty is boiled, then it tastes yucky.*

(b) *Monty tastes yucky if it is boiled.*

(c) *Monty is boiled implies it tastes yucky.*

(d) *Monty is boiled only if it tastes yucky.*

(e) *Monty is not boiled, or it tastes yucky.*
This one uses the logical equivalence between $P \rightarrow Q$ and $\neg P \vee Q$.

(f) *Monty tastes yucky unless it is not boiled.*
Logically, "unless" means the same as "or" (although sometimes it seems to mean "exclusive or").

(g) *Monty tasting yucky is necessary for it being boiled.*
More colloquiallly, perhaps we should say:
For Monty, tasting yucky is a necessary consequence of it being boiled.

(h) *Monty being boiled is sufficient for it tasting yucky.*
More colloquiallly, perhaps we should say:
For Monty, being boiled is a sufficient condition for it tasting yucky.

◦ Implications combined with universal quantification

A universally quantified implication is a statement of the form "for any x, $A(x)$ implies $B(x)$", where x is an element from some domain, and $A(x)$ and $B(x)$ are predicates on that domain. Using mathematical notation, we would write something like

$$\forall x \Big(A(x) \rightarrow B(x) \Big).$$

For our purposes here, we will use the following domain and predicates.

> Let our domain be the set of all cucumbers.
> $A(x)$: x is boiled.
> $B(x)$: x tastes yucky.
>
> **Aside:** Our previous example of $P \to Q$ may be written as $A(\text{Monty}) \to B(\text{Monty})$.

Here are common ways to say $\forall x \Big(A(x) \to B(x) \Big)$.

(a) *For any cucumber x, if x is boiled, then x tastes yucky.*

(b) *For every cucumber x, if x is boiled, then x tastes yucky.*

(c) *For all cucumbers x, if x is boiled, then x tastes yucky.*

(d) *A boiled cucumber tastes yucky.*

(e) *Any boiled cucumber tastes yucky.*

(f) *Every boiled cucumber tastes yucky.*

(g) *Boiled cucumbers taste yucky.*

(h) *All boiled cucumbers taste yucky.*

(i) *Cucumbers taste yucky when they are boiled.*

(j) *A cucumber tastes yucky whenever it is boiled.*

(k) *If cucumbers are boiled, then they taste yucky.*

(l) *A cucumber tastes yucky if it is boiled.*

(m) *Cucumbers are boiled only if they taste yucky.*

(n) *Any cucumber tastes yucky unless it is not boiled.*

(o) *Cucumbers taste yucky unless they are not boiled.*

(p) *Boiling a cucumber is sufficient for it to taste yucky.*
More colloquiallly, perhaps we should say:
Being boiled is a sufficient condition for a cucumber to taste yucky.

(q) *Cucumbers tasting yucky is necessary for them being boiled.*
More colloquiallly, perhaps we should say:
Tasting yucky is a necessary consequence of boiling cucumbers.

(r) *For any cucumber, being boiled implies tasting yucky.*

An observant reader should notice that there are many ways to express the $\forall x$ part, including ones that do not include the word "for". Here are some examples.

- *For any cucumber* $\cdots$
- *For every cucumber* $\cdots$
- *For each cucumber* $\cdots$

- *For all cucumbers* $\cdots$
- *A cucumber* $\cdots$
- *Any cucumber* $\cdots$
- *Every cucumber* $\cdots$
- *Cucumbers* $\cdots$
- *All cucumbers* $\cdots$

There are also many ways to express the $\rightarrow$ part. Here are some examples.

- *if* $\cdots$ *then* $\cdots$
- $\cdots$ *implies* $\cdots$
- $\cdots$ *when* $\cdots$
- $\cdots$ *whenever* $\cdots$
- $\cdots$ *if* $\cdots$
- $\cdots$ *only if* $\cdots$
- $\cdots$ *unless* $\cdots$
- $\cdots$ *sufficient* $\cdots$
- $\cdots$ *necessary* $\cdots$

Furthermore, any way of expressing the $\forall x$ part can be combined with any way expressing the $\rightarrow$ part.

Exercise:
Find many other ways to say *boiled cucumbers taste yucky*.

◦ $\cdots$ **such that** $\cdots$

Too many students seem to use *such that* incorrectly. In mathematics, *such that* is most frequently used to qualify an object that was just introduced. Here are some examples.

- Let x be a real number such that $x^2 + 1 > 5$.
- There exists $p \in \mathbb{N}$ such that both p and $p + 2$ are prime.
- If f is a function such that f is continuous on interval $[0, 1]$, then there is a number M such that $f(x) \leq M$ for any $x \in [0, 1]$.

A good way to see if you have used *such that* correctly is to replace it with *with the property* (or *with the condition*), and see if the meaning of the sentence remains the same. Try making the suggested replacements in the above examples and see how the meaning remains the same.

Here is an example of an incorrect use of *such that*.

"If f is continuous on $[0, 1]$, then f is bounded on $[0, 1]$."

does not have the same meaning as

"f is continuous on $[0, 1]$ such that f is bounded on $[0, 1]$."

Indeed the second sentence does not even make sense because f was not just introduced. What f are we talking about? Perhaps it was introduced in a previous sentence? In this case, replacing *such that* with *with the property* does not result in something that has the same meaning as the first sentence.